Composite FUNctions
Domain and Range :)

From last class:
We determined that the y-value of the inner function in a composite function becomes the x-value of the outer function.

Because of this, the range of the inner function restricts the domain of the outer.

This means we cannot simply look at a composite function and determine its domain and range.
How do we determine the domain and range??

- Determine the domain and range of the inner function.

- The DOMAIN of the composite IS the RANGE of the inner (use x instead of y of course).

- The RANGE of the composite is then determined by subbing its domain restrictions into the outer function.
Example: Given \( f(x) = x + 3 \) and \( g(x) = x^2 - 3 \) 
Determine:

a. \( f(g(x)) = (x^2-3)+3 \)
   \[ = x^2 \]

b. The domain and range of \( f(g(x)) \).

Domain & Range of inside function:
\[ g(x) = x^2 - 3 \]
- \( D: \{ x | x \in \mathbb{R} \} \)
- \( R: \{ y | y \geq -3, y \in \mathbb{R} \} \)

Domain & Range of composite function:
\[ f(x) = x + 3 \]
- Domain is restricted by range of inside function.
  - \( D: \{ x | x \geq -3, x \in \mathbb{R} \} \)
- Range is restricted by domain.
  - Since \( x \geq -3 \), the lowest y-value is:
    - \( f(x) = x + 3 \)
    - \[ = -3 + 3 \]
    - \[ = 0 \]
  - \( R: \{ y | y \geq 0, y \in \mathbb{R} \} \)
Example: Given $f(x) = x + 3$ and $g(x) = x^2 - 3$

Determine:

c. $g(f(x)) = (x + 3)^2 - 3$

d. The domain and range of $g(f(x))$.

**Inside function**

$f(x) = x + 3$

$D: \{x | x \in \mathbb{R}\}$

$R: \{y | y \in \mathbb{R}\}$

**Composite function**

$g(x) = x^2 - 3$

$D: \{x | x \in \mathbb{R}\}$

$R: \{y | y \geq -3, y \in \mathbb{R}\}$
Example: Given $f(x) = |x - 3|$ and $g(x) = -x^2$
Determine:

a. $f(g(x)) = |-x^2 - 3|$

b. The domain and range of $f(g(x))$.
   
   $g(x) = -x^2$
   
   D: $\{x | x \in \mathbb{R}\}$
   
   R: $\{y | y \leq 0, y \in \mathbb{R}\}$

   $f(x) = |x - 3|$
   
   D: $\{x | x \leq 0, x \in \mathbb{R}\}$

   Range:
   
   $f(0) = |0 - 3|$
   
   $= |-3|$
   
   $= 3$

   R: $\{y | y \geq 3, y \in \mathbb{R}\}$
Example: Given \( f(x) = |x - 3| \) and \( g(x) = -x^2 \)
Determine:

c. \( g(f(x)) = -\left| x - 3 \right|^2 \)

d. The domain and range of \( g(f(x)) \).

**Inside Function**
- \( D: \{x \mid x \neq 3\} \)
- \( R: \{y \mid y \geq 0, y \in \mathbb{R}\} \)

**Composite Function**
- \( D: \{x \mid x \geq 0, x \in \mathbb{R}\} \)
- \( R: \{y \mid y \leq 0, y \in \mathbb{R}\} \)
Page 86

#4 - find domain and range of $g(f(x))$ and $f(g(x))$.

#5a

#6a and b

$(fog)(x)=f(g(x))$
Page 86 #4

\[ g(x) = 3x + 2 \quad f(x) = -2x^2 \]

\( \text{d: } \{x \mid x \in \mathbb{R}\} \quad \{x \mid x \in \mathbb{R}\} \)

\( \text{r: } \{y \mid y \in \mathbb{R}\} \quad \{y \mid y \leq 0, y \in \mathbb{R}\} \)

\[ g(f(x)) = 3(-2x^2) + 2 \]

Domain is the same as range of inside function.

D: \( \{x \mid x \leq 0, x \in \mathbb{R}\} \)

Range: Sub restricted x-values into outside function to get y-values.

Outside: \( 3x + 2 = y \)

\( x \leq 0 \)

\( 3(0) + 2 = y \)

\( 2 = y \)

Range: \( \{y \mid y \leq 2, y \in \mathbb{R}\} \)